

Spontaneous regular structure amplification in strongly turbulent rotating fluids

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An initial stage of spontaneous generation of regular structures in strongly turbulent rotating fluids is studied in the framework of the Charney-Hasegawa-Mima model [J. Pedlosky, *Geophysical Fluid Dynamics* (Springer, New York, 1979); A. Hasegawa and K. Mima, *Phys. Fluids* **21**, 87 (1978)]. It is shown that small-scale turbulence may maintain a regular structure in a mean flow. The corresponding growth rate is shown to have a form of negative hyperviscosity. Possible applications in the theoretical study of large scale vortices in the Jovian atmospheres are discussed.

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This paper is devoted to the problem of the formation and sustainment of large-scale ordered Rossby-type structures in strongly turbulent rotating fluids modeling planetary atmospheres. During the last 20 years many theoretical papers were devoted to this problem. A lot of them [1–11] started from the Charney-Hasegawa-Mima (CHM) equation [2,4] that describes both linear and strongly nonlinear vortexlike Rossby waves in a rotating fluid as well as low-frequency drift waves in a magnetized plasma. It was shown that Rossby vortices might have both dipole [5] and monopole [6] structures. Besides analytical research numerous laboratory [7–10] and numerical [11,12] simulations have been carried out. Rossby vortices were shown to be the best model solutions describing strongly nonlinear large-scale objects in atmospheres of planets. But the process of a transformation of well-known linear Rossby waves into the strongly nonlinear vortices still has to be studied. One possibility is a spontaneous generation of large-scale mean flows by turbulent Reynolds stresses, as utilized in the so-called vortex-dynamo model [13–16]. The idea is that the small-scale turbulence may be considered as consisting of a number of high-frequency quanta moving on the background of a mean flow formed by the large-scale motions and acting on the large scales by some pondermotive force. An analogous problem has been studied in paper [17] considering the interaction of regular vortex structures with small-scale weak drift-wave turbulence in magnetized plasmas. It was shown that a so-called “self-organization instability” describing the flow of the energy of turbulent pulsations to the regular vortices could occur. Spontaneous mean shear flow amplification in weakly turbulent plasmas (in the framework of the model incorporating the adiabatic electron response) was also shown to be possible in paper [18]. The interaction of large-scale regular Rossby structures (that were assumed to be already shaped) with weak Rossby turbulence and the influence of the vortices’ existence on two-dimensional (2D) turbulence spectra were studied in papers [19,20].

The present paper considers the interaction of large-scale regular Rossby waves with small-scale Rossby-wave turbulence that, unlike in papers [17–20], is supposed to be strong (the characteristic time of turbulent spectra formation is assumed to be much smaller than the time of a regular structure growth). The interaction is studied with the aid of the direct interaction approximation (DIA). The modified CHM equation, taking into account the influence of small-scale turbulence on regular structure dynamics, is obtained. An instability that explains the large-scale Rossby mode growth due to an interaction with turbulence is shown to be possible. The characteristic size of a vortex is estimated.

It is well known that the dynamics of both linear and nonlinear Rossby waves in a rotating atmosphere may be properly described by a vorticity transport equation (known as the CHM equation) written in the quasigeostrophic β -plane approximation [1,2]:

$$\frac{\partial}{\partial t} (\nabla_{\perp}^2 - r_R^{-2}) \Phi - (\vec{\nabla}_{\perp} \Phi \times \vec{z}) \cdot \vec{\nabla}_{\perp} \nabla_{\perp}^2 \Phi + \beta \frac{\partial \Phi}{\partial x} = \nu_0 \nabla_{\perp}^4 \Phi, \quad (1)$$

where β is the gradient of the Coriolis force, $\Phi = \Phi(\vec{x}_{\perp}, t)$ is the stream function of the two-dimensional flow (that is supposed to be a superposition of turbulent pulsations and a regular mean flow), $\vec{x}_{\perp} = (x, y, 0)$, $\vec{\nabla}_{\perp} \equiv (\partial/\partial x, \partial/\partial y, 0)$, $r_R = \sqrt{gH}/f$ is the Rossby radius, and H , g , f , and ν_0 are the mean vertical depth, gravitational acceleration, twice the angular velocity, and a molecular viscosity, respectively. In this paper we are most interested in structures of the ensemble-mean flow. For this purpose, following paper [17] we separate a stream function Φ into the large-scale ensemble-mean part $\langle \phi \rangle$ (the appearance of which may be, for example, due to the existence of an inverse energy cascade in 2D turbulence [21]) and the small-scale turbulent part Φ^T as $\Phi(\vec{x}_{\perp}, t) = \langle \Phi(\vec{x}_{\perp}, t) \rangle + \Phi^T(\vec{x}_{\perp}, t)$, where the bracket $\langle \rangle$ means averaging over the fast time of small-scale fluctuations (note that $\langle \Phi \rangle$ depends on time through a sequence of ensemble averages made while keeping time fixed). Substituting this expression into Eq. (1) and averaging it in the

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usual manner we obtain the following system of equations for the mean flow and the turbulent field, respectively:

$$\begin{aligned} & \frac{\partial}{\partial t}(r_R^{-2} - \nabla_{\perp}^2)\langle\Phi\rangle - \beta \frac{\partial\langle\Phi\rangle}{\partial x} + \nu_0 \nabla_{\perp}^4\langle\Phi\rangle - [\langle\Phi\rangle, \nabla_{\perp}^2\langle\Phi\rangle] \\ & = \langle[\Phi^T, \nabla_{\perp}^2\Phi^T]\rangle, \end{aligned} \quad (2)$$

$$\begin{aligned} & \frac{\partial}{\partial t}(r_R^{-2} - \nabla_{\perp}^2)\Phi^T - \beta \frac{\partial\Phi^T}{\partial x} + \nu_0 \nabla_{\perp}^4\Phi^T - [\Phi^T, \nabla_{\perp}^2\Phi^T] \\ & = [\Phi^T, \nabla_{\perp}^2\langle\Phi\rangle] + [\langle\Phi\rangle, \nabla_{\perp}^2\Phi^T] - [\Phi^T, \nabla_{\perp}^2\Phi^T], \end{aligned} \quad (3)$$

where $[a, b]$ means the Jacobian

$$[a, b] = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x}. \quad (4)$$

In the presence of a regular structure, the total turbulent field Φ^T may be represented in the form

$$\Phi^T = \Phi^{T(0)} + \Phi^{T(1)}, \quad (5)$$

where $\Phi^{T(0)}$ is the turbulent field in the absence of a mean flow and $\Phi^{T(1)}$ is a turbulence perturbation due to the existence of a mean flow. The latter is supposed to be small, namely $|\Phi^{T(1)}| \ll |\Phi^{T(0)}|$ (a more precise condition will be obtained below). Substituting Eq. (5) into Eq. (3) and restricting ourselves to terms linear over $\Phi^{T(1)}$, we get two equations for $\Phi^{T(0)}$ and $\Phi^{T(1)}$ respectively,

$$\begin{aligned} & \frac{\partial}{\partial t}(r_R^{-2} - \nabla_{\perp}^2)\Phi^{T(0)} - \beta \frac{\partial\Phi^{T(0)}}{\partial x} + \nu_0 \nabla_{\perp}^4\Phi^{T(0)} \\ & - [\Phi^{T(0)}, \nabla_{\perp}^2\Phi^{T(0)}] = 0, \end{aligned} \quad (6)$$

$$\begin{aligned} & \frac{\partial}{\partial t}(r_R^{-2} - \nabla_{\perp}^2)\Phi^{T(1)} - \beta \frac{\partial\Phi^{T(1)}}{\partial x} + \nu_0 \nabla_{\perp}^4\Phi^{T(1)} \\ & - [\Phi^{T(1)}, \nabla_{\perp}^2\Phi^{T(0)}] - [\Phi^{T(0)}, \nabla_{\perp}^2\Phi^{T(1)}] \\ & = [\Phi^{T(0)}, \nabla_{\perp}^2\langle\Phi\rangle] + [\langle\Phi\rangle, \nabla_{\perp}^2\Phi^{T(0)}]. \end{aligned} \quad (7)$$

Equation (6) defines statistical properties of the background turbulence. Here and hereafter we suppose these properties to be given and are interested only in their influence on large-scale motions. In fact, this corresponds to a kinematic approximation. Introducing the Fourier transformation

$$f(\vec{k}; t) = \frac{1}{(2\pi)^2} \int d^2\vec{x} f(\vec{x}, t) \exp(-i\vec{k} \cdot \vec{x}), \quad (8)$$

where $\vec{k} \equiv \vec{k}_{\perp} = (k_x, k_y, 0)$, we rewrite Eqs. (3) and (7) in the following form:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + i\omega_{\vec{q}} \right) \langle\Phi(\vec{q}; t)\rangle + \frac{1}{2(q^2 + r_R^{-2})} \int d\vec{q}_1 d\vec{q}_2 \delta(\vec{q} - \vec{q}_1 - \vec{q}_2) \\ & \quad \times \Lambda_{\vec{q}_1, \vec{q}_2} \langle\Phi(\vec{q}_1; t)\rangle \langle\Phi(\vec{q}_2; t)\rangle \\ & = - \frac{1}{2(q^2 + r_R^{-2})} \int d\vec{k}_1 d\vec{k}_2 \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) \\ & \quad \times \Lambda_{\vec{k}_1, \vec{k}_2} \langle\Phi^T(\vec{k}_1; t)\Phi^T(\vec{k}_2; t)\rangle, \end{aligned} \quad (9)$$

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + i\omega_{\vec{k}} \right) \Phi^{(1)}(\vec{k}; t) \\ & \quad + \frac{1}{k^2 + r_R^{-2}} \int d\vec{k}_1 d\vec{k}_2 \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) \\ & \quad \times \Lambda_{\vec{k}_1, \vec{k}_2} \Phi^{T(0)}(\vec{k}_1; t) \Phi^{T(1)}(\vec{k}_2; t) \\ & = - \frac{1}{k^2 + r_R^{-2}} \int d\vec{q} d\vec{k}_1 \delta(\vec{k} - \vec{q} - \vec{k}_1) \\ & \quad \times \Lambda_{\vec{q}, \vec{k}_1} \langle\Phi(\vec{q}; t)\rangle \Phi^{T(0)}(\vec{k}_1; t), \end{aligned} \quad (10)$$

where $\Lambda_{\vec{k}_1, \vec{k}_2} = \vec{z} \cdot (\vec{k}_1 \times \vec{k}_2) (k_1^2 - k_2^2)$ is the well-known matrix element of Rossby wave interactions and $\omega_{\vec{k}} = -\beta k_x / (k_{\perp}^2 + r_R^{-2}) - i\nu_0 k_{\perp}^4 / (k_{\perp}^2 + r_R^{-2})$ is the Rossby wave frequency. Note that \vec{q} stands for the wave vector of a large-scale motion while \vec{k} means the wave vector of small-scale turbulence. The solution of Eq. (10) may be chosen in the following form:

$$\begin{aligned} \Phi^{T(1)}(\vec{k}; t) & = - \frac{1}{k^2 + r_R^{-2}} \int d\vec{q} d\vec{k}_1 \delta(\vec{k} - \vec{q} - \vec{k}_1) \Lambda_{\vec{q}, \vec{k}_1} \\ & \quad \times \int_{-\infty}^t dt_1 \hat{G}(\vec{k}; t - t_1) \langle\Phi(\vec{q}; t_1)\rangle \Phi^{T(0)}(\vec{k}_1; t_1), \end{aligned} \quad (11)$$

where $\hat{G}(\vec{k}; t - t_1)$ is the Green's function of Eq. (10) [$\langle\hat{G}\rangle = G(\vec{k}; t - t_1)$]:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + i\omega_{\vec{k}} \right) \hat{G}(\vec{k}; t - t_1) + \frac{1}{k^2 + r_R^{-2}} \int d\vec{k}_1 d\vec{k}_2 \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) \\ & \quad \times \Lambda_{\vec{k}_1, \vec{k}_2} \Phi^{T(0)}(\vec{k}_1; t) \hat{G}(\vec{k}_2; t - t_1) = \delta(t - t_1). \end{aligned} \quad (12)$$

We assume the basic turbulent field $\Phi^{T(0)}(\vec{k}; t)$ to be homogeneous and isotropic:

$$\langle\Phi^{T(0)}(\vec{k}; t)\Phi^{T(0)}(\vec{k}'; t')\rangle = \delta(\vec{k} + \vec{k}') Q(\vec{k}; t - t'), \quad (13)$$

where $Q(k; t - t')$ means the correlation function of the basic field. Substituting expression (11) along with Eq. (5) into the right-hand side of Eq. (9), averaging it by using Eq. (13), and neglecting the nonlinear self-action of the regular structure (this linearization procedure is valid when the condition

$q_1^2 \langle \phi \rangle / \omega_{\vec{q}} \ll 1$ is satisfied), we obtain the following equation for $\langle \Phi(\vec{q}, t) \rangle$:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + i\omega_{\vec{q}} \right) \langle \Phi(\vec{q}, t) \rangle \\ &= \frac{1}{4\pi(q^2 + r_R^{-2})} \int d\vec{k}_1 \vec{z} \cdot (\vec{k}_1 \times \vec{q}) [2(\vec{k}_1 \cdot \vec{q}) - q^2] \\ & \times \int_{-\infty}^t dt_1 \int d\omega_1 d\omega_2 d\omega_3 \delta(\omega_1 - \omega_2 - \omega_3) \\ & \times \left(G(\vec{q} - \vec{k}_1; \omega_2) Q(\vec{k}_1, \omega_3) \frac{\Lambda_{\vec{q}, -\vec{k}_1}}{(\vec{q} - \vec{k}_1)^2 + r_R^{-2}} \right. \\ & \left. + G(\vec{k}_1; \omega_2) Q(\vec{q} - \vec{k}_1; \omega_3) \frac{\Lambda_{\vec{q}, \vec{k}_1 - \vec{q}}}{k_1^2 + r_R^{-2}} \right) \\ & \times \exp[-i\omega_1(t - t_1)] \langle \Phi_{\vec{q}}(t_1) \rangle, \end{aligned} \quad (14)$$

where $G(\vec{k}; \omega)$ and $Q(\vec{k}; \omega)$ are Fourier images in time of the values $G(\vec{k}; \tau)$ and $Q(\vec{k}; \tau)$ ($\tau = t - t_1$), respectively:

$$\{G(\vec{k}; \omega); Q(\vec{k}; \omega)\} = \frac{1}{2\pi} \int d\tau \{G(\vec{k}; \tau); Q(\vec{k}; \tau)\} \exp(i\omega\tau). \quad (15)$$

In the case of strong turbulence these values may be obtained from the basic equations (6) and (12) with the aid of the DIA. Following the papers [22,23], we assume simple functional forms involving a few coefficients for G and Q that, in the framework of a kinematic approximation, are considered as given ones. Namely, we suppose them to be defined by the following expressions:

$$\begin{aligned} -i(\omega - \omega_{\vec{q}}) &= \frac{1}{2i(q^2 + r_R^{-2})} \int d\vec{k}_1 \vec{z} \cdot (\vec{k}_1 \times \vec{q}) (2\vec{k}_1 \cdot \vec{q} - q^2) \left\{ \Lambda_{\vec{q}, -\vec{k}_1} \left(\frac{1}{\omega - \omega_{\vec{q}-\vec{k}_1} - \omega_{\vec{k}_1} + i\Omega(\vec{k}_1) + i\lambda\Omega(\vec{q}-\vec{k}_1)} \frac{w(\vec{k}_1)}{(\vec{q}-\vec{k}_1)^2 + r_R^{-2}} \right. \right. \\ & \left. \left. - \frac{1}{\omega - \omega_{\vec{q}-\vec{k}_1} - \omega_{\vec{k}_1} + i\lambda\Omega(\vec{k}_1) + i\Omega(\vec{q}-\vec{k}_1)} \frac{w(\vec{q}-\vec{k}_1)}{k_1^2 + r_R^{-2}} \right) + \frac{\vec{z} \cdot (\vec{q} \times \vec{k}_1) (2\vec{k}_1 \cdot \vec{q} - q^2) w(\vec{q}-\vec{k}_1)}{[\omega - \omega_{\vec{q}-\vec{k}_1} - \omega_{\vec{k}_1} + i\lambda\Omega(\vec{k}_1) + i\Omega(\vec{q}-\vec{k}_1)] (k_1^2 + r_R^{-2})} \right\}. \end{aligned} \quad (19)$$

In obtaining Eq. (19), we employed no assumption about the characteristic scale of the regular structure change. Now we suppose that this scale greatly exceeds that of the turbulent pulsations, i.e., the inequality $|\vec{q}| \ll k_c$ is assumed to be satisfied, where k_c is the minimum value of wave numbers characterizing the energy-containing small-scale turbulent motion, which is related to the largest size l of turbulent energy-containing eddies as $k_c = 2\pi/l$. We seek the solution of Eq. (19) in the form $\omega = \omega_{\vec{q}} + \Delta\omega_{\vec{q}} + i\gamma_N$ ($\gamma_N \ll \omega_{\vec{q}}$,

$$G(\vec{k}; \omega) = \frac{1}{-i\omega + i\omega_{\vec{k}} + \lambda\Omega(\vec{k})}, \quad (16)$$

$$Q(\vec{k}; \omega) = \frac{w(\vec{k})}{i\pi[\omega - \omega_{\vec{k}} + i\Omega(\vec{k})]}, \quad (17)$$

where $w(\vec{k})$ defines the spectral energy density, and $\Omega(\vec{k})$ is the eddy-turnover frequency defining the time of turbulent spectra formation (in the weak turbulence theory it is usually considered as a small correction to the linear kernel defined by $\omega_{\vec{k}}$). Let us assume that a well-developed inertial range does exist. Then, according to the Kolmogorov hypothesis, the value $\Omega(\vec{k})$ should be defined [23] by the expression $\Omega(\vec{k}) = \alpha \varepsilon^{1/3} |\vec{k}|^{2/3}$ (α is some numerical factor and ε is a turbulent energy dissipation rate). Here λ is a number, $\lambda \sim 1$. It would be equal to 1 exactly if the value $G(\vec{r}; t - t_1)$ was defined by the following equation for the Green's function of Eq. (6):

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + i\omega_{\vec{k}} \right) \hat{G}(\vec{k}; t - t_1) + \frac{1}{2(k^2 + r_R^{-2})} \int d\vec{k}_1 d\vec{k}_2 \delta(\vec{k} - \vec{k}_1 \\ & - \vec{k}_2) \Lambda_{\vec{k}_1, \vec{k}_2} \Phi^{T(0)}(\vec{k}_1; t) \hat{G}(\vec{k}_2; t - t_1) = \delta(t - t_1) \end{aligned} \quad (18)$$

instead of Eq. (12).

Substituting Eqs. (16) and (17) into Eq. (14), and performing the Fourier transformation in time, we obtain the following dispersion equation for Rossby waves in the case considered:

$\Delta\omega_{\vec{q}} \ll \omega_{\vec{q}}$. In this paper we also suppose that $(qr_R)^2 \ll 1$ and $\gamma_N \ll \Omega(k_c)$ (i.e. the characteristic time of regular structure growth is supposed to greatly exceed the characteristic time of turbulent spectra formation). The opposite case of weak turbulence, when the condition $\gamma_N \gg \Omega(k_c)$ is satisfied, was considered in paper [17] [note that in that case Eq. (19) reduces to Eq. (78) of paper [17] by neglecting terms $\propto q^2/k_1^2$]. In the case considered here Eq. (19) can be rewritten in the following form:

$$\begin{aligned}
\gamma_N - i\Delta\omega_{\vec{q}} \approx & \frac{2r_R^4}{1+\lambda} \int d\vec{k}_1 \frac{[\vec{z} \cdot (\vec{k}_1 \times \vec{q})]^2 (\vec{k}_1 \cdot \vec{q})}{(1+k_1^2 r_R^2)^2 \Omega(k_1)} \\
& \times \left\{ \vec{q} \cdot \frac{\partial h}{\partial \vec{k}_1} + (1+k_1^2 r_R^2) q^2 \left(\vec{q} \cdot \frac{\partial w}{\partial \vec{k}_1} \right) \right. \\
& - \left(q^2 - ik_1^2 \frac{\Delta\omega_{\vec{q}} + i\gamma_N + \omega_{\vec{q}} - \vec{q} \cdot \vec{v}_{g\vec{k}_1}}{(1+\lambda)\Omega(k_1)} \right) \\
& \left. \times \left(\vec{q} \cdot \frac{\partial}{\partial \vec{k}_1} \frac{h}{k_1^2} \right) \right\}, \quad (20)
\end{aligned}$$

where the group velocity is defined as $\vec{v}_{g\vec{k}_1} = d\omega_{\vec{k}_1}/d\vec{k}_1$, $w = w(k_1)$, $h = h(k_1)$, and

$$h(k) = \frac{1}{2} k^2 (1 + k^2 r_R^2) w(k) \quad (21)$$

is the spectral enstrophy density. Supposing that value $\Omega_1(\vec{k})$ is defined by its inertial-range form and performing the integration in the right-hand side of Eq. (20) over the angle variable δ , where the vector \vec{k}_1 is defined as $k_{x1} = k_1 \cos(\delta)$, $k_{y1} = k_1 \sin(\delta)$, we obtain the following expressions for the nonlinear frequency shift and nonlinear growth rate of a regular Rossby structure in the case we consider here:

$$\Delta\omega_{\vec{q}} \approx \frac{\pi\omega_{\vec{q}}^4 r_R^4}{6(1+\lambda)^2} \int_{k_c}^{\infty} dk_1 \frac{k_1^5 (7 + k_1^2 r_R^2) w(k_1)}{(1+k_1^2 r_R^2)^2 \Omega^2(k_1)}, \quad (22)$$

$$\gamma_N = \gamma_1 q^4 - \gamma_2 q^6, \quad (23)$$

$$\begin{aligned}
\gamma_1 \approx & \frac{\pi r_R^4}{6(1+\lambda)} \left(r_R^2 \int_{k_c}^{\infty} dk_1 \frac{k_1^7 w(k_1)}{(1+k_1^2 r_R^2)^2 \Omega(k_1)} \right. \\
& \left. - 5 \int_{k_c}^{\infty} dk_1 \frac{k_1^5 w(k_1)}{(1+k_1^2 r_R^2)^2 \Omega(k_1)} \right), \quad (24)
\end{aligned}$$

$$\gamma_2 \approx \frac{4\pi r_R^4}{3(1+\lambda)} \int_{k_c}^{\infty} dk_1 \frac{k_1^3 w(k_1)}{(1+k_1^2 r_R^2) \Omega(k_1)}. \quad (25)$$

We chose the low limit of an integral in the right-hand side of Eqs. (22)–(25) in the form of “ k_c ” instead of “0” in order to be consistent with the assumption that the turbulence is small scale. The “self-organization instability” resulting in the turbulent energy cascade to the regular structures may obviously occur in the case when $\gamma_1 > 0$. According to Eqs. (23)–(25), the most unstable regular mode has the wave number $q_{\max} = (2\gamma_1/3\gamma_2)^{1/2}$. In order to be self-consistent with the previous study it must satisfy the inequality $q_{\max} \ll k_c$. The self-organization instability cannot take place in the case when $k_{\max}^2 r_R^2 \ll 1$, where k_{\max} is the maximum value of the turbulent motion wave number (in this case the value γ_1 is negative). It may not also be predicted in the opposite case $k_c^2 r_R^2 \gg 1$ [when the second term in the right-hand side of Eq. (24) may be neglected] because, despite the fact that in this case $\gamma_1 > 0$, the characteristic length scale of the most unstable mode turns out to be comparable to that of

the turbulence. The latter is not consistent with the scale separation procedure used in this paper. The self-organization instability of the large-scale Rossby structures due to their interaction with strong Rossby-wave turbulence may be anticipated for some intermediate situation when $k_c r_R \sim 1$, and both terms in the right-hand side of Eq. (24) are of the same order. This process may be stipulated by the fact that the regular structures enhance the inhomogeneity of the turbulence (that was initially homogeneous); meanwhile the inhomogeneous part of the turbulence affects the regular structures. In other words, we have feedback [17]. It would of course be fine to have a physical explanation for the possibility of such a process for some regimes of 2D turbulence and impossibility for others.

An analysis shows that our assumption of $\gamma_N \ll \Omega(k_c)$ is justified as long as we have small-scale turbulence, i.e., $q_{\perp}^2/k_c^2 \ll 1$. In order to be consistent with the previous assumption $|\Phi^{T(1)}| \ll |\Phi^{T(0)}|$, the inequality $(k_c \langle \Phi \rangle) / [L\Omega(k_c)] \ll 1$ should be satisfied, where L is the characteristic scale of a regular Rossby structure. This condition may be violated when a regular structure contains sheared background flows, which may significantly affect the stability issue [24], so this case needs special consideration, which will be carried out in the near future.

To illustrate the results obtained, we consider the problem of spontaneous amplification and sustainment of large vortices like the Great Red Spot in the turbulent Jovian atmospheres. According to observation data [10–12], the Jovian atmospheric motions that may be considered as small-scale Rossby waves forming a turbulent spectrum have length scales in the range $\sim 10^3$ – 10^4 km, while the Rossby radius is $r_R \sim 3 \times 10^3$ km. To evaluate a characteristic length scale of a large-scale structure that may be generated in such a system, we have to calculate the integrals in Eqs. (24) and (25). We note that the observed spectra of Rossby and drift-wave turbulence (in the Hasegawa-Mima model) vary as $w(k) \propto 1/k^6$ for $kr_R \sim 1$ [25,26]. Such a distribution can be understood as the broadest spectrum consistent with a logarithmically convergent value for the enstrophy as k_{\max} tends to infinity. In this case, simple calculations give the following length of the regular structure: $L = 2\pi/q_{\max} \sim 3.96 \times 10^4$ km. This result is in good correspondence with the observed characteristic scale of the Great Red Spot of Jupiter: $L \approx 2.5 \times 10^4$ km.

In this paper we have considered the stability of a large-scale regular Rossby structure interacting with strong small-scale Rossby-wave turbulence. Starting from the Charney-Hasegawa-Mima equation, we studied the problem of whether a coherent regular structure could grow in such a system. We separated the stream function into regular and turbulent components. Equation (14), together with expressions (16) and (17), were derived representing a consistent model describing the influence of small-scale turbulence on regular structure dynamics. As a result of this model, we were able to detect the self-organization instability, i.e., the growth of the regular Rossby structures due to turbulent fluctuations, and to study its linear stage. The nonlinear stage of Rossby-wave interactions with turbulence including the very formation of nonlinear structures is now being considered for different types of structures, both analytically and numerically, and will be presented in the near future.

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